IV. Conclusion

A leaky waveguide with extremely low losses is proposed for long-distance transmission of submillimeter waves, and its guiding mechanism is fully analyzed by using a two-dimensional slab waveguide model. It is also shown that long-distance transmission is possible in submillimeter through optical wavelengths. Attenuation constants of the leaky TE_{0n} and TM_{0n} modes in round structure are also presented.

The excitation efficiency of the leaky modes and additional losses due to bends will be analyzed in future publications.

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A Multilayer Fiber Guide with Rectangular Core

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***Abstract*—An approximate analysis is given for multilayer dielectric waveguides with rectangular core. Dispersion curves are calculated for several modes over a range of parameters. Some comments related to the design of such guides for use with planar structures such as semiconductor lasers are given.**

FIBER GUIDES with rectangular cores are important for compatibility with many integrated optics devices to which they may be connected [1]–[3]. Although exact analysis of the rectangular geometry has not been done, the available approximate analyses [4], [5], [14] are useful, provided there is a single cladding material. There may, however, be advantage in using two or more layers with different indices, as shown in Fig. 1. Planar and circular cylindrical versions of the multilayer dielectric guides have shown advantages in propagation

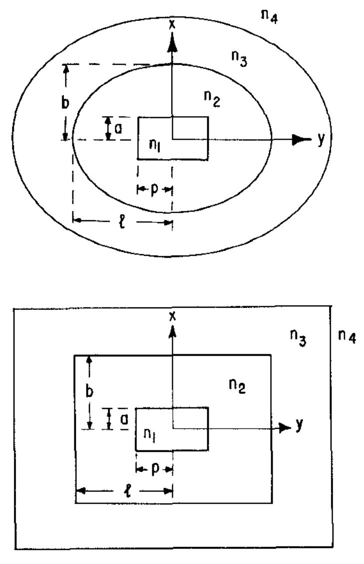

Fig. 1. (a) Fiber guide in cross section. (b) Basic model for analysis.

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characteristics and mode control when indexes of the several layers are properly selected [6], [7], [8]. In this letter we utilize Marcatili's approach [4] to the rectangular geometry to obtain propagation characteristics of such multilayer guides with rectangular core. Marcatili's method has been successfully used for many other purposes (see, for example, [5], [9], [10]).

Marcatili's method applies continuity conditions to the four sides of the rectangle independently, not worrying about the inconsistency of this approach at the external corner regions. It is reasonable that this should give useful results, at least in the region of greatest usefulness when fields are largely confined to the core. Comparisons with other approaches have shown that this is so. The basic model is that of Fig. 1(b), but since the corners are of little importance, results should apply also to the cross section of Fig. 1(a). But in the case of Fig. 1(a) we have to specify that curvature of the interface between mediums 2 and 3 does not change the essential structure of guided mode fields. In other words, the curvature of that interface should be much larger than the wavelength, or more correctly, Shevchenko's inequalities [11] $b \geqslant 2k_3^2/k_{x_3}^3$ and $l \geqslant 2k_3^2/k_{y_3}^3$ should be satisfied where $k_3 = 2\pi n_3/\lambda$, λ is the wavelength in free space k_{x_3} and k_{y_3} are the wavenumbers along axes x and y, respectively, in the layer with n_3.

The region 3 is assumed thick enough so that any external protective cladding n_4 does not affect the solution. Thus we are considering only three-region problems. We adopt the division into E_{mn}^x modes with E_x and H_y as the main transverse components, and H_{mn}^x modes, stressing E_y and H_x, as in Marcatili [4]. But, since optical guides are characterized by small differences between the refractive indices $|n_1 - n_2| \ll 1$ and $|n_3 - n_2| \ll 1$, the propagation characteristics for the two classes differ by less than one part in 10^4 so that numerical results given apply to either class within the accuracy of plotting.

For guided modes propagating in the z direction the appropriate product solutions are sinusoids or exponents. Continuity of tangential fields applied across each boundary individually (as explained above) then yields the characteristic equation. We give this for the individual cases.

Case 1

$n_1 > n_2 > n_3$ (for Fig. 2(a)) (this special case has been treated in [11], [12]) and $n_2 > n_1 > n_3$ (Fig. 2(b)). For boundary conditions assumed the characteristic equation for H_{mn}^x modes becomes a system of two transcendental equations:

$$x_1\psi_1(x_1\psi_1 \tan 2cx_1 + x_2\psi_2) + x_2\psi_2(x_1\psi_1 - x_1\psi_1 \tan 2cx_1) = 0$$

$$y_1\phi\{y_1\phi_1 \tan 2qy_1 + [(n_1/n_2)^2 - (cx_1/\pi a')^2] y_2\phi_2\} + [(n_1/n_2)^2 - (cx_1/\pi a')^2] y_2\phi_2 \cdot \{y_1\phi_1 - [(n_1/n_2)^2 - (cx_1/\pi a')^2] y_2\phi_2 \tan 2qy_1\} = 0$$

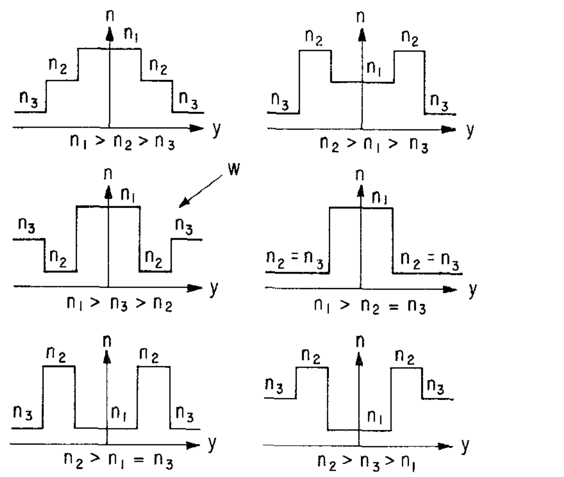

Fig. 2. Refractive index distributions in the cross section of fiber guides.

where

$$\psi_1 = x_2 + x_3 \tan(1-c)x_2$$

$$\psi_2 = x_2 \tan(1-c)x_2 - x_3$$

$$\phi_1 = y_3 \tan(1-q)y_2 + (n_3/n_2)^2 y_2$$

$$\phi_2 = (n_3/n_2)^2 y_2 \tan(1-q)y_2 - y_3$$

and $c = a/b$, $q = p/l$, $x_1 = k_{x_1}b$, $x_2 = k_{x_2}b$, $x_3 = k_{x_3}b$, $y_1 = k_{y_1}l$, $y_2 = k_{y_2}l$, $y_3 = k_{y_3}l$ are dimensionless transverse wavenumbers, a' is the relative thickness of the rectangular core,

$$a' = 2an_2/\lambda = c(x_1^2 + x_3^2)^{1/2}/\pi[(n_1/n_2)^2 - (n_3/n_2)^2]^{1/2}$$

$\lambda_2 = \lambda/n_2$, and λ is the wavelength in the free space. Moreover, the wavenumbers are related by

$$[(n_1/n_2)^2 - (n_3/n_2)^2]x_2^2 = [1 - (n_3/n_2)^2]x_1^2 - [(n_1/n_2)^2 - 1]x_3^2$$

and

$$[(n_1/n_2)^2 - (n_3/n_2)^2 - (cx_1/\pi a')^2]y_2^2 = [1 - (n_3/n_2)^2]y_1^2 - [(n_1/n_2)^2 - 1 - (cx_1/\pi a')^2]y_3^2.$$

It is clear that the phase constant β lies in the range $n_3/n_2 \leqslant \beta/k_2 \leqslant n_1/n_2$ for $n_1 > n_2 > n_3$ and in the range $n_3/n_2 \leqslant \beta/k_2 \leqslant 1$ for $n_2 > n_1 > n_3$, where the left-hand equality sign corresponds to the cutoff conditions and the right-hand sign to conditions far from the cutoff, and $k_2 = 2\pi n_2/\lambda$.

Case 2

$n_1 > n_3 > n_2$ (W-fiber guide, Fig. 2(c)) and $n_1 > n_2 = n_3$ (Fig. 2(d)), and also for the case $n_1 > n_2 > n_3$ (Fig. 2(a)) when $x_3[(n_1/n_2)^2 - 1] > x_1[1 - (n_3/n_2)^2]$ and $[(n_1/n_2)^2 - 1 - (cx_1/\pi a')^2]y_3 > [1 - (n_3/n_2)^2]y_1$.

In these cases of refractive index distribution in the cross section the wavenumbers x_2 and y_2 become imaginary ($x_2 = jx_2, y_2 = jy_2$) and the characteristic equation reduces to the next system of two equations

$$x_1\psi_1(-x_1\psi_1\tan 2cx_1 + x_2\psi_2) + x_2\psi_2(x_1\psi_1 - x_2\psi_2\tan 2cx_1) = 0$$

$$y_1\phi_1\{-y_1\phi_1\tanh 2qy_1 + [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2\} + [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2 \cdot \{y_1\phi_1 - [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2\tanh 2qy_1\} = 0$$

where

$$\psi_1^* = x_2 + x_3\tanh(1-c)x_2$$
$$\psi_2^* = x_2\tanh(1-c)x_2 + x_3$$
$$\phi_1^* = y_3\tanh(1-q)y_2 + (n_3/n_2)^2 \cdot y_2$$
$$\phi_2^* = (n_3/n_2)^2 y_2\tanh(1-q)y_2 + y_3.$$

The wavenumbers are related by

$$[(n_1/n_2)^2 - (n_3/n_2)^2]x_2^2 = [(n_1/n_2)^2 - 1]x_3^2 - [1 - (n_3/n_2)^2]x_1^2$$

and

$$[(n_1/n_2)^2 - (n_3/n_2)^2 - (cx_1/\pi a')^2]y_2^2 = [(n_1/n_2)^2 - 1 - (cx_1/\pi a')^2]y_3^2 - [1 - (n_3/n_2)^2]y_1^2.$$

The phase constant lies in the range $n_3/n_2 \leqslant \beta k_2 \leqslant n_1/n_2$ for the $n_1 > n_3 > n_2$ and $n_1 > n_2 > n_3$, and in the range $1 \leqslant \beta/k_2 \leqslant n_1/n_2$ for the $n_1 > n_2 = n_3$, and in the range $1 \leqslant \beta/k_2 \leqslant n_1/n_2$ for the $n_1 > n_2 = n_3$. The relative thickness of the rectangular core is expressed through the wavenumbers as in the first case.

Case 3

$n_2 > n_1 = n_3$ (Fig. 2(e)) and $n_2 > n_3 > n_1$ (Fig. 2(f)).

For such a type of cross-sectional index distributions the wavenumbers x_1 and y_1 become imaginary ($x_1 = jx_1, y_1 = jy_1$), and the characteristic equation can be represented as the system of two equations:

$$x_1\psi_1(-x_1\psi_1\tan 2cx_1 + x_2\psi_2) + x_2\psi_2(x_1\psi_1 - x_2\psi_2\tan 2cx_1) = 0$$

$$y_1\phi_1\{-y_1\phi_1\tanh 2qy_1 + [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2\} + [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2 \cdot \{y_1\phi_1 - [(n_1/n_2)^2 + (cx_1/\pi a')^2]y_2\phi_2\tanh 2qy_1\} = 0$$

where ψ_1, ψ_2, ϕ_1, ϕ_2 are the same as in the first case. The wavenumbers are related by

$$[1 - (n_3/n_2)^2]x_1^2 = [(n_3/n_2)^2 - (n_1/n_2)^2]x_2^2 + [1 - (n_1/n_2)^2]x_3^2$$

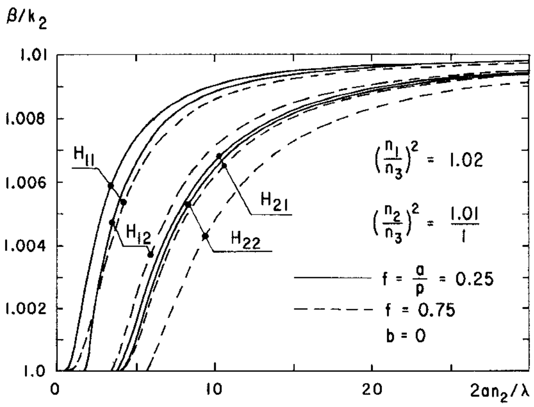

Fig. 3. Dispersion curves for the limiting case without the intermediate layer ($b = 0$) of the fiber guide for different thickness ratios a/p of the core.

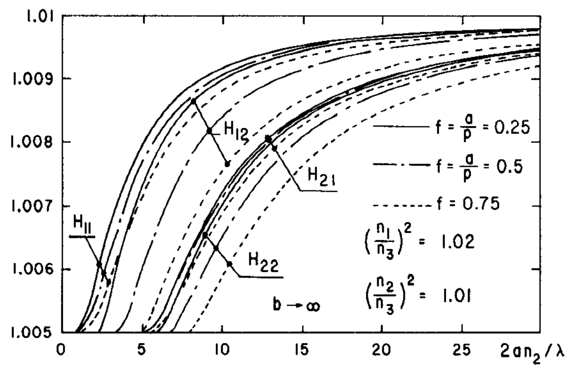

Fig. 4. Dispersion curves for the limiting case when the thickness of the intermediate layer of the fiber guide is equal to infinity ($b = \infty$) for different thickness ratios a/p of the core.

and

$$[1 - (n_3/n_2)^2]y_1^2 = [1 - (n_1/n_2)^2 - (cx_1/\pi a')^2]y_3^2 + [(n_3/n_2)^2 - (n_1/n_2)^2 - (cx_1/\pi a')^2]y_2^2.$$

The relative thickness of the rectangular core a' is determined as $a' = 2an_2/\lambda = c(x_2^2 + x_3^2)^{1/2}/\pi[1 - (n_3/n_2)^2]^{1/2}$. The range for the phase constant is $n_3/n_2 \leqslant \beta/k_2 \leqslant 1$ for both index distributions.

The dispersion curves for different types of the cross-sectional index distribution of the multilayer fiber guides with the rectangular core are presented in Figs. 3–6. The expression for the normalized phase constant can be deduced in terms of the dimensionless wavenumbers as

$$\frac{\beta}{k_2} = \sqrt{\left(\frac{n_1}{n_2}\right)^2 - \frac{(cx_1)^2 + (qfy_1)^2}{(\pi a')^2}}\ .$$

Comparative analysis of the six types of fiber guides which differ by the cross-sectional index distribution has demonstrated that the fiber guide of W type with $n_1 > n_3 > n_2$ has the largest core transverse thickness, larger than these if fiber guides with considered distributions of the refractive index. Fig. 3 is the limiting case without the

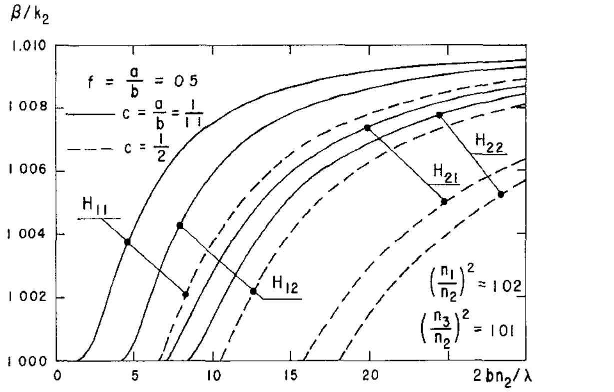

Fig. 5. Dispersion curves for the fiber guide with rectangular core (thickness ratio of the core $a/p=0.5$) for different thickness of the intermediate layer.

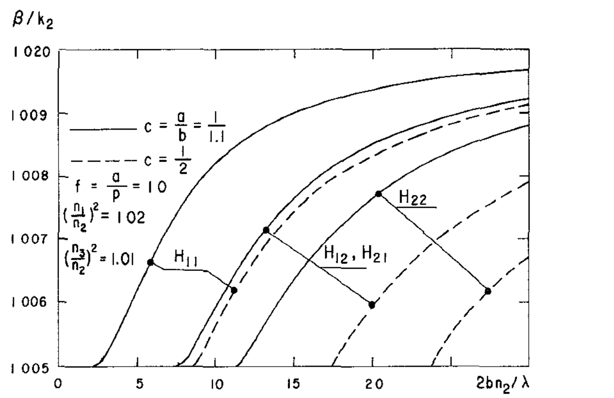

Fig. 6. Dispersion curves for the fiber guide with square core (thickness ratio of the core $a/p=1$) for different thicknesses of the intermediate layer.

intermediate region ($b=0$), and Fig. 4 is the limiting case with only two media ($b=\infty$). In both cases the values of β/k_2 at cutoff and far above cutoff are as expected.

Conclusion

It is shown that it is possible to calculate the dispersion characteristics of multilayer fiber guides with the rectangular core using the classical method of Marcatili and Shevchenko's condition of similarity between a curved and planar dielectric waveguide. Some design features for multilayer fiber guides with the rectangular core are also discussed.

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